# Real Money Trade in Online Videogame Market as a Primary Source of Income.

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## Introduction

The world is constantly changing and new technologies and activates are introduced. For instance, fifty years before videogames were not a cultural phenomenon as they are now, and the online videogames did not exist at all. The games are highly addictive because of the amount of rewards that you get in them and some individuals would want to spend all their time playing videogames. There is a barrier to that type of life, however; a person has basic human needs, such as shelter and food, for which real money is spent. Where from a person who spends all the time playing videogames gets the real money to pay for the needs? The first thing that comes to mind would be for such person to have saving of money that would be enough for the lifetime. It is a rare occasion, obviously. However, another source of income now exists: the game themselves. Playing an online game with many other player, a person accumulates online goods that he or she can sell to other players who would want to buy.

With the degree of massive online games, such as MMORPG World of Warcraft (WoW) which once had 12 million subscribers in 2010, such situations become even more possible (Number of World of Warcraft subscribers from 1st quarter 2005 to 4th quarter 2013 (in millions), 2014). The acronym MMORPG stands for Massive Multiplayer Online Role Playing Game – a game where you control a character that becomes better as you play (RPG part) and that is happening on online servers with lots of players playing simultaneously in the same game world, so called "realm"(MMO part). The game has its in-game currency for that amount of players to interact with; however, consider online markets, where players trade in-game currency with the real world-currency and selling in-game characters, again, for the real money. The prices can go phenomenal: for instance, one WoW character was sold for around $9,000! (Hill, 2012). However, there should be somebody with real money to buy such goods; therefore, the people who play the games all the time need some players who have real-life job from which they could get real-life money.

The gaming companies sometimes are not happy about the real money trade and the real money markets for the characters and other in-game goods are usually grey. Thus, it would be worthwhile to explore whether the real money trade (RMT) makes the players better off than if they would be without trade – that is, if a complete ban on the RMT would make the players better off.

The approach, and the model to some extent, used in this paper would be available to apply to other contexts where people can trade something that they acquire. For instance, the community of bug collectors. However, it is difficult to find a buyer for your production, if you are living in a small city and the only one interested in bugs. Geographical location is not an issue for the online good market: people from all around the world can see your trades and buy them with almost immediate delivery. Therefore, this framework is applicable to some other situations, but relies on unique features of online gaming, such as big community and instant delivery.

## Literature Review

As the massive online games are relatively new, there is an insignificant amount of research done on that regards, especially with developing an abstract economic model. The existing research involved in virtual trade mostly focuses on the virtual goods from a consumer's perspective.

In their study "Price convergence in an online virtual world", Morrison and Fontenla are trying to determine whether the rule of one price applies to World of Warcraft (Morrison & Fontenla, 2013). By using datasets from 8 different webservers, the researchers have concluded that the prices of in-game items in different in-game fractions converge on all the servers tested. In addition, the researchers have noted that online games often create "abstract versions" of real world, similar to economic models. That study is a vivid example on how economic tools, namely, the rule of one price, can be tested in virtual realm. However, the study does not look on the interactions of virtual and real markets.

In the article "'Gaming is my work': identity work in internet-hobbyist game workers", the researchers Lee and Lin are looking on the psychological and working conditions of game workers – the people who make real money by selling virtual goods (Lee & Lin, 2011). Noting that such phenomenon is called "real money trade" (RMT) and by conducting interviews, the researchers concluded that the workers can still suffer from poor working conditions and have troubles with telling a work from a hobby since a game is both at the same time for them. Although this study focuses on the real money trade workers, it is qualitative in nature and more belongs to the field of psychology than to economics.

The study conducted by Constantiou et al, expressed in article "What are users' intentions towards real money trading in massively multiplayer online games?", tries to test the factors that make players participate in RMT (Constantiou, Legarth, & Olsen, 2012). Constantiou et al conducted a questionnaire among World of Warcraft players and came to several conclusions, including that the perceived in-game social status is important to players and thus would positively influence the probability that a player would participate in RMT. Note that the approach made by the researchers belongs to the realm of behavioral economics and they were not looking on the market at the whole.

It becomes clear from the literature that there the researchers approach online videogames seriously and from different perspective: macroeconomic, psychological and from behavioral economics.

# Theoretical Model

## Real Side

In the model, we will have a pool of individuals who will have an opportunity to work and to trade.

First, we are assuming that everyone wants to play and have a good character. Although that may look not so realistic, but remember that we are only considering the individuals who play the games and participate in trade.

In addition, we are getting a population of players from one game to eliminate a choice between games and to have one trade market.

However, we should not forget that the people want to satisfy their everyday needs and buy good goods for the real world as well. Therefore, the people are also getting utility from their real-world disposable income:

$$U_i = f(disposable\ income, in - game\ pleasure)$$

As another simplifying assumption, the savings are not allowed and the only sources of income would be the person's job and whatever the person got as a result of trading in-game goods. Assuming that there is only one market for trading in-game goods with endogenous market price $p$ (since it is an online market consisting of only players so the price adjusts) and that each individual has a pay-by-hour job yielding $w_i$ (revenue less cost), the following holds:

$$disposable\ income = w_i * e_{wi} + p * q_{ti},$$

where $e_{wi}$ is the effort spent on working and $q_{ti}$ is the balance of trade of individual $i$. That is, the balance of trade is positive when the individual sells the in-game goods and negative when the individual buys in-game goods. Note that amount of time spent on working is assumed to be the same as the effort.

In addition to the effort spent on working - $e_{wi}$, we also introduce the amount of effort spent on playing the game - $e_{pi}$.

Further note, the effort cannot be limitless; otherwise, the model may not be interesting and realistic. Consider the following condition:

$$e_{wi} + e_{pi} \leq Et,$$

where $Et$ is the maximum amount of effort that an individual spends. The number $Et$ is limited by the number of hours in a day or a number of working hours – that is, a number of hours in a day less time for sleeping and eating. In addition, assume that any unused hours from $Et$ are wasted without affecting any other variables including utility; say, the person is just passively watches TV or browses the internet yielding 0 utility per unit of effort spent.

## Virtual Side

Let us now talk about the virtual side of an individual's life. As noted before, an individual $i$ uses $e_{pi}$ of effort for playing and produces some amount of virtual goods during that time denoted as $g_i$. Let us assume a linear production function: $g_i = f(e_{pi}) = e_{pi}$. Such function would be a nice depiction of a so-called *grinding* – a repetitive process that you have to encounter during the game. Leveling up a character or obtaining in-game currency often involves collecting items or exterminating many enemies.

We further assume that the cost associated with grinding balances out the utility gained (Grinding (video gaming), n.d.). That is reasonable, as that process of grinding is mundane. In addition, it corresponds well with Lee and Lin research, as playing for selling is confusingly between working and playing (Lee & Lin, 2011). However, the players are getting some utility from the character that they have in the end of the day. That nature of utility of the character also reflects the proven importance of in-game social status (or character quality) for the players (Constantiou, Legarth, & Olsen, 2012). The following would hold:

$$character\ quality = g_i - q_{ti},$$

as selling in-game goods would decrease the quality of character (and buying goods would increase the quality). The terms "in-game goods", "virtual goods" and "character quality" will be used interchangeably.

Note that an essential part of playing online games is interplayer interaction. In a case of role-playing games, the players can compete against each other so a player with better character quality would be better off. Therefore, to show the competitive nature of the game,

$$in - game\ utility = \frac{character\ quality}{average\ quality\ of\ other\ players} = \frac{g_i - q_{ti}}{Q_{-i}},$$

where $Q_{-i}$ precisely defined as an average quality of a non-$i$ player's character.

Thus, a consumer utility would be characterized by the following equation:

$$U_i = f(disposable\ income, in - game\ pleasure) = f\left(w_i * e_{wi} + p * q_{ti}, \frac{g_i - q_{ti}}{Q_{-i}}\right).$$

## Further Functional Forms

Population would be normalized to a continuum from $0$ to $1$; that is, $i \in [0,1]$. In order to diversify the resulting market so to get more realistic results, $w_i$ should not be constant over $i$. For simplicity, we propose a following uniform distribution:

$$w_i = i * w_{max},$$

with $w_{max}$ being a positive constant. Note that the formula also normalizes the minimum wage to $0$.

Further assumed the following Cobb-Douglas functional form for the utility of an individual:

$$U_i = (disposable\ income)^{\alpha}(in - game\ pleasure)^{\beta} = (w_i * e_{wi} + p * q_{ti})^{\alpha}\left(\frac{g_i - q_{ti}}{Q_{-i}}\right)^{\beta},$$

for some constants $\alpha, \beta \in (0,1]$. Cobb-Douglas form is a nicely-differentiable form which has the property of diminishing utility of components. Thus, an individual would prefer a mix of disposable income and in-game pleasure, which is happening in the real world.

Also note that assuming something about the relationship between $\alpha$ and $\beta$, such as $\alpha + \beta = 1$ does not influences the implications of the analysis much, but can greatly reduce the complexity of derivations. However, both $\alpha$ and $\beta$ are used throughout the paper as a single constant $\beta$ is associated easier than the expression $(\alpha - 1)$ with how much individuals like gaming.

As we have assumed that the market is big, each individual would be a price-taker in the virtual goods market and would expect to have zero effect on the other players' average quality; that is, take $p$ and $Q_{-i}$ as given.

# Theoretical Results

## Solution for the model with trade

To begin with, we need to solve an individual's problem. Recall that each individual has the following utility:

$$U_i = (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{g_i - q_{ti}}{Q_{-i}}\right)^{\beta}.$$

Substitution the linear production function $g_i = e_{pi}$ and the effort constraint $e_{wi} + e_{pi} \leq Et$ (which is equivalent to $e_{wi} + e_{pi} = Et$ since increasing effort always increases utility) yields the following individual's utility:

$$U_i = (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta}.$$

Where the only variables that the individual has the control of are $e_{wi}$ and $q_{ti}$. Maximizing the utility subject to the constraints yields the following corner solutions (for full derivation, see appendix 1):

$$\begin{cases} \begin{cases} e_{wi} = 0 \\ q_{ti} = \dfrac{\alpha}{\alpha + \beta} Et' \end{cases} & if\ w_i < p \\ \begin{cases} e_{wi} = Et \\ q_{ti} = \dfrac{\beta}{\alpha + \beta} \dfrac{-w_i * Et}{p}, \end{cases} & if w_i > p \end{cases}.$$

We can see that the society has divided into two categories of people, who do not work and sell in-game items and the people who work maximum allowed number of hours, but buy in-game goods. We can call those groups "gamers" and "workers". Note that "gamers" are the people with lower $w_i$, therefore, $i$.

Next, we need to find the equilibrium price. Since the demand must meet supply, the market clearing condition would be

$$\int_0^1 q_{ti}\, di = 0,$$

since $q_{ti}$ is a balance of trade of individual $i$ and $i \in [0,1]$. Denoting the share of "gamers" $\gamma$ and using the assumption that $\alpha + \beta = 1$, fact that $w_i = i * w_{max}$ and an observation that $w_{\gamma} = \gamma * w_{max} = p$ (an individual indifferent between being a "worker" and a "gamer") and that $Q_{-i} = average\ quality \equiv Q$ (since the number of individuals is large), the integration results in

$$p = w_{max} \sqrt{\frac{\beta}{2\alpha + \beta}};\ \ \gamma = \sqrt{\frac{\beta}{2\alpha + \beta}}.$$

As for $Q$, total amount of in-game items produced is $\gamma * Et$ (each of the $\gamma$ "gamers" plays $Et$) and the population size of yields

$$Q = \frac{1}{1} * Et * \gamma = Et\sqrt{\frac{\beta}{2\alpha+\beta}}.$$

Substitution of the solutions to the individual problem to the utility function yields

$$U_i = \begin{cases} EtQ^{-\beta}\alpha^{\alpha}\beta^{\beta}p^{\alpha}, & if\ i < \gamma \\ EtQ^{-\beta}\alpha^{\alpha}\beta^{\beta}w_i p^{-\beta}, & if\ i > \gamma \end{cases}.$$

Or, with the substituted equilibrium values,

$$U_i = \begin{cases} Et^{\alpha}\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \alpha^{\alpha}\beta^{\beta}(w_{max})^{\alpha}, & if\ i < \sqrt{\frac{\beta}{2\alpha+\beta}} \\ Et^{\alpha}(2\alpha+\beta)^{\beta}\alpha^{\alpha} * i * w_{max}^{\alpha}, & if\ i > \sqrt{\frac{\beta}{2\alpha+\beta}} \end{cases}$$

A rough visualization of utility distribution is the following:

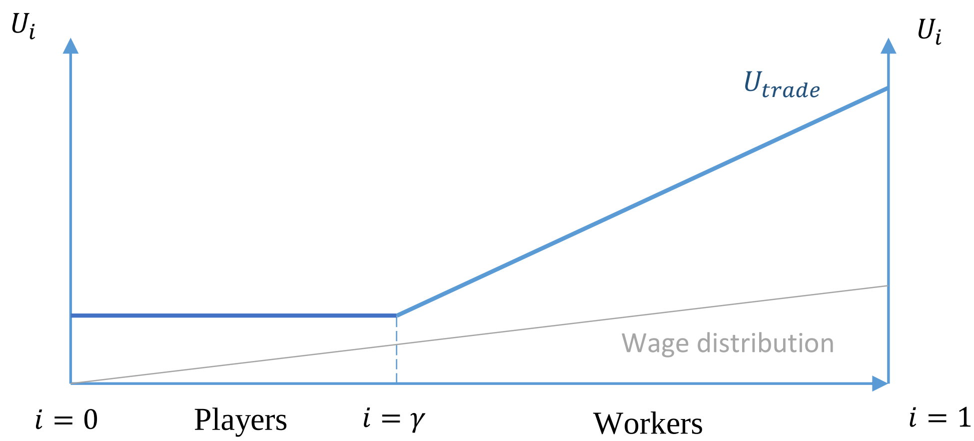


*Figure 1 Utility distribution for the problem with trade*

Note that the gamers have a flat utility. That happens because they chose to play all the time. However, that does not mean that they do not care about their wage – it is the flat price provided by the market. In addition, if they see that their wage is greater than $w_{\gamma}$, they switch to being a worker so they do consider their wage. As for the workers, their utility function is linearly increasing in $i$ so is a line. Notice, however, that they are getting diminishing marginal utility from their linearly increasing income so the expected shape is a square root-like. However, they use their increase in income to buy virtual goods as well so those two effects on utility – from increase of income and from buying virtual goods – overlap and result in linear relationship.

It is also possible to calculate the total utility of the population by integrating the $U_i$'s over $i$. So,

$$TU = \int_0^1 U_i\, di = Et^\alpha \alpha^\alpha w_{max}^\alpha (2\alpha + \beta)^{-\alpha}.$$

## Solution the model with a trade ban

However, the government or the game company might ban trade. In that case, each $q_{ti} = 0$ and an individual's utility function takes form

$$U_i' = (w_i * e_{wi})^\alpha \left(\frac{Et - e_{wi}}{Q_{-i}}\right)^\beta,$$

which has only one variable of choice - $e_{wi}$. Maximizing by taking derivate yields

$$e_{wi}{}' = \frac{\alpha}{\alpha + \beta} Et.$$

Therefore, individual's decision does not depend on his or her wage – everyone puts the same amount of effort for work and for playing. That happens partially because the wage factors out during the process because of the functional form chosen.

We can also calculate $Q'$ - the average character quality. Since everyone puts the same amount of effort in playing,

$$Q' = Et - e_{wi}' = \frac{\beta}{\alpha + \beta} Et.$$

One can further substitute $e_{wi}{}'$ and $Q'$ values into the individual's utility function and get the following form:

$$U_i{}' = \left(w_i * \frac{\alpha}{\alpha + \beta} Et\right)^\alpha.$$

Note that everybody would get 1 from playing the game since all individuals are at the average character quality; the function reflects that in absence of the virtual utility ($\left(\frac{Et - e_{wi}}{Q_{-i}}\right)^\beta$).

Note the form of the utility function: the only thing that varies is $w_i$. It, in turn has only variance in $i$ (as $w_i = i * w_{max}$). Therefore, the function takes form $C * i^\alpha$. As $\alpha \in [0,1]$, the distribution would be represented by something that looks like a square root function, linear when $\alpha = 0$. That happens because of diminishing marginal utility of money.

Here is a visual representation of the utility distribution:

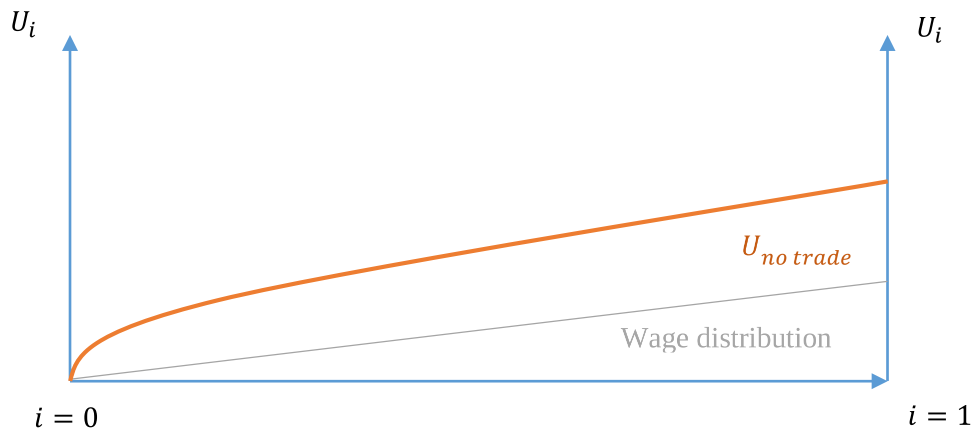


*Figure 2 Utility distribution for the problem without trade*

It is also relatively simple to calculate the overall utility:

$$TU' = \int_0^1 U_i \, di = \frac{1}{2\alpha + \beta}\left(w_{max}\frac{\alpha}{\alpha + \beta}Et\right)^{\alpha}.$$

## Comparison of the solutions for the two versions

Now let us compare the results with and without trade. Let us compare the utility values for significant individuals and the values of $Q$ and $TU$. Assuming $\alpha + \beta = 1$, it can be shown that under $\alpha > 0$, which means that the players are not indifferent about their real lives, that

$$\begin{aligned} U_{trade,0} &> U_{no\ trade,0} \\ U_{trade,\gamma} &< U_{no\ trade,\gamma} \\ U_{trade,1} &> U_{no\ trade,1} \\ Q_{trade} &> Q_{no\ trade} \\ TU_{trade} &> TU_{no\ trade}. \end{aligned}$$

Using this information, it is possible to make the following figure:

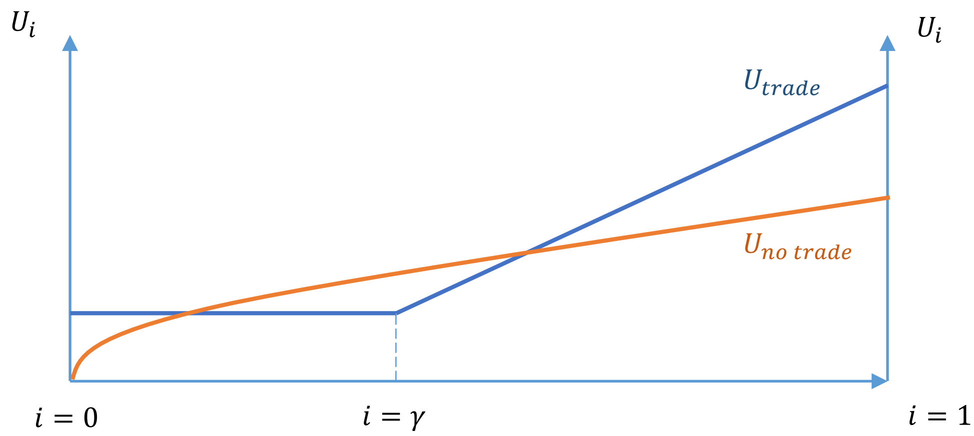


*Figure 3 Both utility distributions plotted*

Thus, the marginal people are better off with trade, while the relatively middle individuals are actually worse off. That makes sense as the 0th individual, the one that has $w_0 = 0$ is definitely better off as without trade it would get utility of 0. The most productive people (1st individual, for instance) have very high wage but have to spend their effort making their character better if they cannot trade.

In the meantime, trade makes some individuals worse off – if they fall somewhere in the middle individual, $\gamma$. It seem to be counterintuitive as we at first think of trade affecting the extreme individuals. There is a logical explanation for the phenomenon, however. That happens because with the trade, more character quality is produced so the people are actually making a rat race with the other individuals. They could be the same or better off if they produced less character quality but they all take $Q$ as given so they care only about their quality. Everyone wants to be above the $Q$ so they increase their character quality and get to a slightly undesirable equilibrium.

However, the version with trade also provides bigger amount of overall utility, not speaking of the low-income individuals having way higher values of utility because playing for trade provides them with way higher income.

It is also useful to make a graph to see who are better off with and without trade, having $\alpha$ as a variable. The following figure, with the people who are better off shown as colored section, can be plotted:

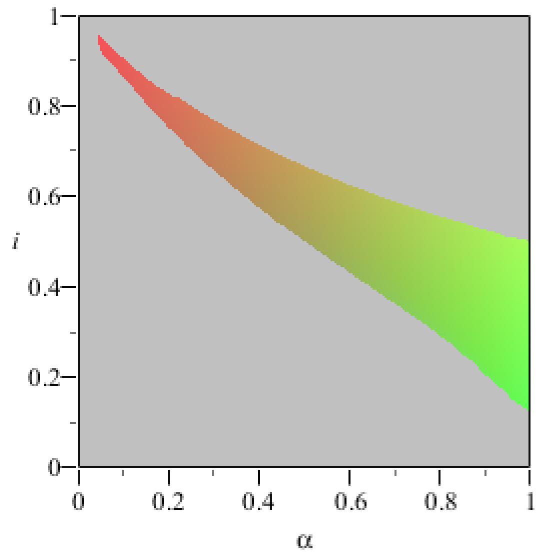


*Figure 4 Individuals who are better off without trade (colored)*

As we can see, the fraction of people who benefit from ban of trade increases as people value real goods more. That happens as when people prefer real goods more, there would be way less game quality produced in the case of the ban of the trade (that effect is explored in Appendix 3: Numeric Solutions). Therefore, people would want to break out from the quality rat race that happens when high-income individuals get more game quality as it is cheap. Notice that as $\alpha$ increases, the more low-income individuals are benefitting from the ban; that is observable from the colored region to move to the lower values of $i$. Also, note that for no values of $\alpha$ there are more than 50% of people who are benefited from the ban.

That effect from switching from trade to no trade can also be shown as a graph of differences of utilities of individuals under trade and under no trade, that is, $U_i - U_i{}'$:

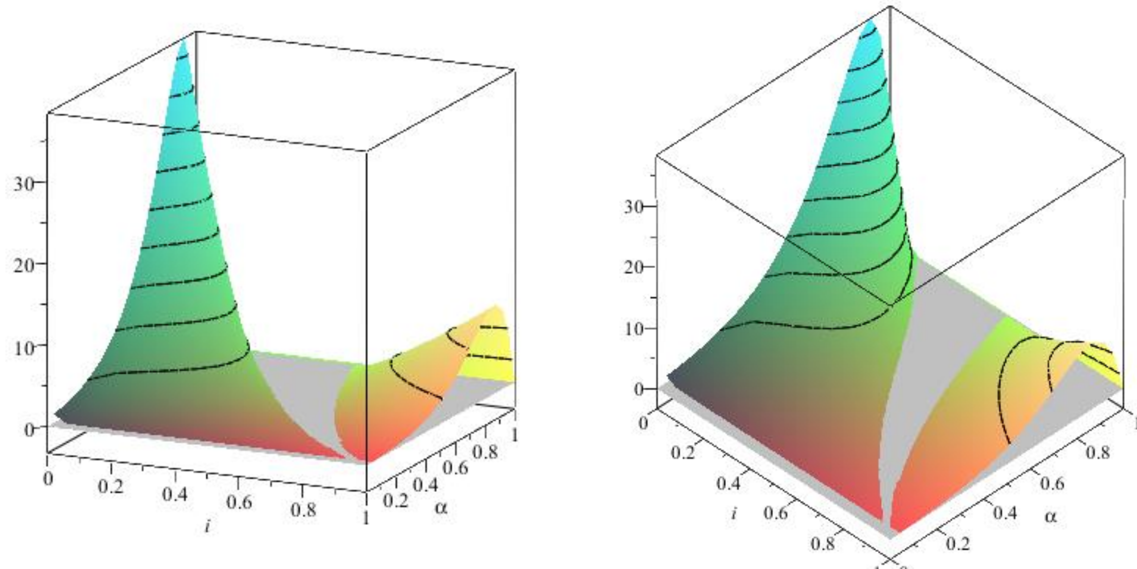


*Figure 5 Difference of utility between trade and no trade for values of $i$ and $\alpha$*

In this plot for $Et = 0$, $w_{max} = 37.97$ (as in Appendix 3: Numeric Solutions), the grey plane is a $U = 0$, that is, $U_i - U_i{}'$. The colored regions are the regions when $U_i > U_i{}'$. As it is possible to see, the extreme individuals – very low and high income – are benefiting from trade way more for the higher values of $\alpha$ as the graph is higher, especially the low-income ones. At the same time, many individuals would prefer no trade for the high values of $\alpha$ as noted before. If the real values are valued more, so $\alpha$ is high, the low-income individuals suffer a lot if there is no trade. Therefore, the price is lower and wealthy high-income individuals can trade more so be happier with trade. In addition, we can see the influence of income distribution: when $\alpha$ is low, the individuals do not care much about their difference in income so being a gamer pays off. On the other hand, when $\alpha$ is high, the difference in wages really matters so there would be more workers and the gamers would agree to sell virtual goods for lower prices.

As analysis had shown, the model suggests that the ban of trade is highly questionable. However, is the model realistic? We are forcing everyone to play – they had no ability to jump out from the game and invest in, for instance, education to increase their wage. In addition, we have used uniform distribution for the wages which is not the case in real life. Too many individuals with high income might have biased the price up so in reality the high-income individuals might get even more utility and the least productive ones would be a bit worse off – but still better than without trade. In addition, the model had not generated the individuals who are mixing their effort in the problem with trade; however, it is more vivid to see the situation with just two corner solution groups.

## Conclusion

Thus, we have constructed an economic model for a self-adjusting market of virtual goods in an online game. The model generated an equilibrium where the population spitted into two groups: "gamers" who have a low opportunity cost of obtaining and selling virtual goods due to low income so they spend all their effort for farming (obtaining) the virtual items; and "workers" who have a high income, work all allowed time and pay the "gamers" for the in-game goods. By doing that, the high-income individuals were saving their time for their productive work while gaining a good character; the "gamers" also enjoy that symbiotic relationship. In addition, we have shown that the complete ban of the trade hurts most of the individuals regardless of the exogenous variables. Interestingly, the marginal individuals were suffering from the ban while a group with the income close to price was getting higher utility when the ban is introduced – the average level of character in the version with trade was always higher. Moreover, it was noticed that the more people care about the real world, the bigger difference in utility happens between the trade and non-trade scenario. That was happening partially because there were differences in income so when there was no trade, the individuals have no way out but to work more and to get less utility if their income is low. While when there was a bias towards the game, everyone would have an equal opportunity to get the same level of characters even if there was no market. In addition, the model is subject to extensions, such as introducing the gaming company to influence the virtual goods market. That was explored in "Appendix 4: Extension: The Gaming Company enters the virtual market" when a greedy profit-maximizing firm ended up monopolizing the supply of the market and forcing everybody to but the virtual goods. It also explores an interesting concept of companies having an ability to create virtual goods costless. However, the companies might create virtual good inflation if they do so too much (and, probably, treat them as fiat money), as in the Appendix 4. In general, the model produces some counterintuitive at first results and is an example of one of few (if any) theoretic market-equilibrium oriented modes for real money trade in an online game, when one can pay others for their skill and time which creates a negative externality on everyone else in the market through the average character level.

The research, however, assumed just one functional form for the utility and for the income distributions. Thus, one can see whether the results hold when changing to different form, for instance, a summation of radicals for the utility and normal distribution for the income. In addition, the model arises other potential directions: making the individuals not to participate in the game, having the players to have different skill level, considering markets of two or more games, etc. However, one of the main directions to follow might be investigating a risk associated with participation in trade: some of the gaming companies, even Blizzard, the developers of WoW, do not allow real money trade so the markets are grey. There is a risk associated with the trade as the company can ban you and your buyer – that is, remove the virtual goods and restrict the access to the game. One more direction of this research could be trying to see how the model fits the real world as it was developed only abstractly and was not checked on real data. In addition, it is more to discuss about the online games, and videogames in general, as that area of research is only arising. For instance, there are moral philosophical questions – is it good for a person to spend all day in the virtual environment? Is that good for the society? Which society – real or virtual one? The presence of virtual spaces and our involvement in them becomes stronger with every day. Due to the digital technology, our world is changing and new problems arise so the science has to take on those issues and situations. Making an abstract economic model for a real money trade in a videogame is a question that had no chance of arising 30 years ago.

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# Appendix 1: Full Solution of Problem with Trade

## Solution for individual's maximization problem

To begin with, we need to solve an individual's problem. Recall that each individual has the following utility:

$$U_i = (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{g_i - q_{ti}}{Q_{-i}}\right)^{\beta}.$$

In the equation, the individual takes $w_i$, $p$, $Q_{-i}$, $\alpha$ and $\beta$ as given, which lives $e_{wi}$, $q_{ti}$ and $g_i$ as the variables of choice. Substitution from the linear in-game goods production function yields

$$U_i = (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{e_{pi} - q_{ti}}{Q_{-i}}\right)^{\beta}.$$

Recall that $e_{wi}$ and $e_{pi}$ are related by $e_{wi} + e_{pi} \leq Et$. However, can there be a case when the left hand side is smaller? In that case, an individual would increase $e_{wi}$ and get higher utility. Following that logic, in equilibrium it would be always the case that $e_{wi} + e_{pi} = Et$. Solving for $e_{pi}$ and substituting to the $U_i$ equation yields

$$U_i = (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta}.$$

Since $Et$ is a constant, a consumer has only two variables of choice left: $e_{wi}$ and $q_{ti}$. Thus, a rational utility-maximizing individual's problem is

$$\max_{e_{wi},q_{ti}} U_i = (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta}.$$

Taking the first derivatives and setting them to 0:

$$\begin{cases} \dfrac{\delta U_i}{\delta e_{wi}} = \alpha w_i (w_i * e_{wi} + p * q_{ti})^{\alpha-1} \left(\dfrac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta} - \dfrac{1}{Q_{-i}} \beta (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\dfrac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta-1} = 0 \\ \dfrac{\delta U_i}{\delta q_{ti}} = \alpha p (w_i * e_{wi} + p * q_{ti})^{\alpha-1} \left(\dfrac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta} - \dfrac{1}{Q_{-i}} \beta (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\dfrac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta-1} = 0 \end{cases}.$$

Subtracting the equations:

$$\begin{cases} \alpha w_i (w_i * e_{wi} + p * q_{ti})^{\alpha-1} \left(\dfrac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta} - \dfrac{1}{Q_{-i}} \beta (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\dfrac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta-1} = 0 \\ \alpha (w_i - p)(w_i * e_{wi} + p * q_{ti})^{\alpha-1} \left(\dfrac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta} = 0 \end{cases}.$$

We know that $\alpha$ is non-zero and that having $w_i * e_{wi} + p * q_{ti}$ or $\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}$ equal to zero would make $U_i$ equal to zero. Therefore, the only possible solution is

$$\begin{cases} \alpha w_i (w_i * e_{wi} + p * q_{ti})^{\alpha-1} \left(\dfrac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta} - \dfrac{1}{Q_{-i}} \beta (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\dfrac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta-1} = 0 \\ (w_i - p) = 0 \end{cases}.$$

Let us simplify the top equation:

$$\begin{cases} \alpha w_i (w_i * e_{wi} + p * q_{ti})^{\alpha-1} \left(\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta} = \frac{1}{Q_{-i}} \beta (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta-1} \\ (w_i - p) = 0 \end{cases}$$

$$\begin{cases} \frac{\alpha}{\beta} w_i Q_{-i} (w_i * e_{wi} + p * q_{ti})^{\alpha-1-\alpha} \left(\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta-(\beta-1)} = 1 \\ (w_i - p) = 0 \end{cases}$$

$$\begin{cases} \frac{\alpha}{\beta} w_i Q_{-i} = \frac{(w_i * e_{wi} + p * q_{ti})}{\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}} \\ (w_i - p) = 0 \end{cases}$$

$$\begin{cases} \frac{\alpha}{\beta} w_i = \frac{(w_i * e_{wi} + p * q_{ti})}{Et - e_{wi} - q_{ti}} \\ (w_i - p) = 0 \end{cases}$$

Since the original equation is in regular Cobb-Douglas form, that is a maxima.

Therefore, the solution appears to be

$$\begin{cases} \frac{\alpha}{\beta} w_i = \frac{Q_{-i}(w_i * e_{wi} + p * q_{ti})}{Et - e_{wi} - q_{ti}} \\ w_i = p \end{cases}.$$

However, that would mean that an individual should set $w_i = p$ while having no control over $w_i$ and $p$! Let us think about that condition. Having $w_i = p$ would mean that an individual is having the same return from a unit of effort spend on working ($w_i$) or on playing and selling the virtual goods ($p$ since the production function is linear). However, there would be only one individual as $w_i$ values are unique. All the other individuals would have either higher or lower $w_i$ so they would strongly prefer either pure work or pure playing and selling. Therefore, there should be a corner solution.

Recall that there are two variables for an individual's choice: $e_{wi}$ and $q_{ti}$, an effort spend on working and the amount of virtual goods traded (bought) correspondingly. We also know that $e_{wi}$ is constrained by $e_{wi} + e_{pi} = Et$. As the effort cannot be negative, both $e_{wi}$ and $e_{pi}$ are greater or equal to zero. Therefore, $e_{wi} \in [0, Et]$.

The amount of virtual goods traded – $q_{ti}$ – is also constrained. First of all, as there are no savings, an individual cannot sell what was not earned. According to the production function, $g_i = e_{pi}$, which with $e_{wi} + e_{pi} = Et$ implies that $\max q_{ti} = g_i = Et - e_{wi}$. The minimum amount of $q_{ti}$ would be when a person spends all the real-world income on buying virtual goods on the market. The amount of goods bought would be $\frac{w_i * e_{wi}}{p}$, thus making $\min q_{ti} = -\frac{w_i * e_{wi}}{p}$. (Recall that buying goods expresses as negative value of $q_{ti}$, the balance of trade.)

Combining the restrictions of the choice variables yields

$$\begin{cases} e_{wi} \in [0, Et] \\ q_{ti} \in \left[-\frac{w_i * e_{wi}}{p}, Et - e_{wi}\right] \end{cases},$$

which, in turn, can be represented on the following diagram:

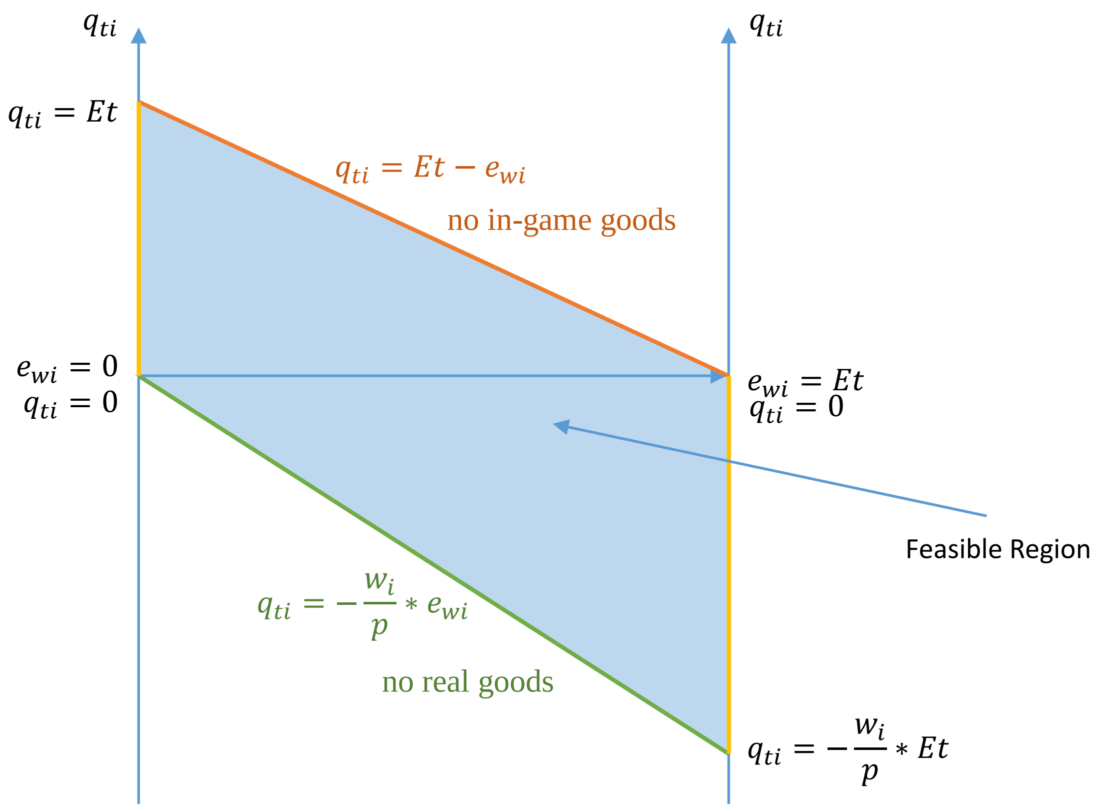


*Figure 6 Feasible region for an individual's decision*

Note, however, that the extreme values of $q_{ti}$ (top and bottom boundaries on the graph) would never give a maximum utility. In the case of maximum value of $q_{ti}$ an individual would sell all the in-game goods and, in the end, get an awful character, having zero utility. In the other case, the individual would spend all the real-world money so left without food and shelter – one more undesirable outcome. Thus, one should expect to have the corner solutions only for the cases $e_{wi} = 0$ or $e_{wi} = Et$ and those would interior for $q_{ti}$.

If an individual choses $e_{wi} = 0$, the following would be his or her problem:

$$\begin{cases} \max_{e_{wi}, q_{ti}} U_i = (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta}, \\ e_{wi} = 0 \end{cases}$$

or

$$\max_{q_{ti}} U_i = (p * q_{ti})^{\alpha} \left(\frac{Et - q_{ti}}{Q_{-i}}\right)^{\beta} = p^{\alpha} \left(\frac{1}{Q_{-i}}\right)^{\beta} q_{ti}^{\alpha} (Et - q_{ti})^{\beta}.$$

Look on the first derivative and simplify:

$$\frac{dU_i}{dq_{ti}} = p^{\alpha} \left(\frac{1}{Q_{-i}}\right)^{\beta} \left(\alpha q_{ti}^{\alpha-1} (Et - q_{ti})^{\beta} - \beta q_{ti}^{\alpha} (Et - q_{ti})^{\beta-1}\right) = 0$$

$$\alpha q_{ti}^{\alpha-1} (Et - q_{ti})^{\beta} = \beta q_{ti}^{\alpha} (Et - q_{ti})^{\beta-1} \qquad (since\ not\ p^{\alpha}\ nor\ \left(\frac{1}{Q_{-i}}\right)^{\beta}\ can\ be\ zero)$$

$$\frac{\alpha}{\beta} q_{ti}^{\alpha-1-\alpha} (Et - q_{ti})^{\beta-(\beta-1)} = 1$$

$$\frac{\alpha}{\beta} = \frac{q_{ti}}{Et - q_{ti}}$$

Note that we got a simplified condition from the two-variable maximization. Let us further solve for $q_{ti}$:

$$\frac{\alpha}{\beta} = \frac{q_{ti}}{Et - q_{ti}}$$

$$\alpha Et - \alpha q_{ti} = \beta q_{ti}$$
$$(\alpha + \beta) q_{ti} = \alpha Et$$
$$q_{ti} = \frac{\alpha}{\alpha + \beta} Et$$

Therefore, one corner solution is

$$\begin{cases} e_{wi} = 0 \\ q_{ti} = \frac{\alpha}{\alpha + \beta} Et \end{cases}$$

Let us look now on what happens if $e_{wi} = Et$:

$$\begin{cases} \max_{e_{wi},q_{ti}} U_i = (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta} \\ e_{wi} = Et \end{cases}$$

And that entails

$$\max_{q_{ti}} U_i = (w_i * Et + p * q_{ti})^{\alpha} \left(\frac{-q_{ti}}{Q_{-i}}\right)^{\beta}.$$

Taking the first derivative gives

$$\frac{dU_i}{dq_{ti}} = \alpha p (w_i * Et + p * q_{ti})^{\alpha-1} \left(\frac{-q_{ti}}{Q_{-i}}\right)^{\beta} - \beta (w_i * Et + p * q_{ti})^{\alpha} \left(\frac{-q_{ti}}{Q_{-i}}\right)^{\beta-1}$$

$$\frac{\alpha}{\beta} p = \frac{w_i * Et + p * q_{ti}}{-q_{ti}}.$$

Solving for $q_{ti}$:

$$\frac{\alpha}{\beta} p(-q_{ti}) = w_i * Et + p * q_{ti}$$

$$-w_i * Et = q_{ti}\left(p + \frac{\alpha}{\beta} p\right)$$

$$q_{ti} = \frac{-w_i * Et}{p\left(1 + \frac{\alpha}{\beta}\right)} = \frac{-w_i * Et}{p\left(\frac{\alpha + \beta}{\beta}\right)} = \frac{\beta}{\alpha + \beta} \frac{-w_i * Et}{p}$$

Thus the second corner solution is

$$\begin{cases} e_{wi} = Et \\ q_{ti} = \frac{\beta}{\alpha + \beta} \frac{-w_i * Et}{p}. \end{cases}$$

Now we have two corner solutions, but when do they happen? Recall that $w_i = p$ causes the individual to be indifferent between working and playing and selling because the real-world money return is the same from those activates. If $w_i < p$, the individual is always better off by playing and selling; therefore, he or she would do $e_{wi} = 0$ so fall into the first corner solution. On the other hand, $w_i > p$ would mean that the amount of working should be maximized; thus, $e_{wi} = Et$ and the individual falls into the second corner solution.

Combining both results together, we get a solution to an individual's problem (in addition, it is possible to show that $q_{ti}$ values fall into the feasible region):

$$\begin{cases} \begin{cases} e_{wi} = 0 \\ q_{ti} = \dfrac{\alpha}{\alpha+\beta} Et' \end{cases} & if\ w_i < p \\ \begin{cases} e_{wi} = Et \\ q_{ti} = \dfrac{\beta}{\alpha+\beta} \dfrac{-w_i * Et}{p}, \end{cases} & if w_i > p \end{cases}.$$

Note that it does not include the case of $w_i = p$. If that happens, as the two-variable maximization tells us, the individual is indifferent. However, there would be only one individual with $w_i = p$ and his or her decision would not be that significant for the market. (On the other hand, it is also very difficult to be the "middle" individual as if you make a tiny deviation – an error, for instance, - that would affect the price a tiny bit and you will not be the middle individual anymore.) Thus, we are further ignoring the decision of such individual.

Further note that having such a piece-wise solution to an individual problem is an interesting one. We have two types of individuals: the ones with their wage lower than the market price and the ones with their wage higher than the market price. The first type never works ($e_{wi} = 0$) and sells positive amount of in-game goods, while the second spend all the effort on working in a job and buys the goods from the market. That would mean that there would be "gamers" and "workers" in the market, correspondingly. Moreover, the types would be divided by some wage $w_{bondary} = p$, which we need to know to talk about the proportion of "gamers" and "workers".

## Market solution

As we have solved the individual's problem, we might look on the market now. The trade market is perfectly competitive so the price would adjust in order to clear the market. In other words, if there were an excess supply of goods on the market, the players with unsold goods would set lower price, driving the overall price down and vice versa. Therefore, in equilibrium there would be no excess supply or demand so if we sum up all the individuals balances of trades, which represents selling or buying, the result should be zero:

$$\int_0^1 q_{ti}\, di = 0.$$

However, it would we very hard to solve that without any further simplifications. Let us remember that there are many players on the market and individuals would not matter in the big picture. Thus, we can say that for all $i$, $Q_{-i} = Q$, where $Q$ is the average quality of characters across the entire population.

Now assume that $\gamma \in (0,1)$ is the proportion of the first type of individuals, "gamers". In that case, we can split the integration:

$$\int_0^\gamma q_{ti}\, di + \int_\gamma^1 q_{ti}\, di = 0,$$

so the first part would be the sellers and the second one would be buyers.

Substituting the individual's solution and the wage distribution $w_i = i * w_{max}$ yields

$$\int_0^\gamma \frac{\alpha}{\alpha+\beta} Et\, di + \int_\gamma^1 \frac{\beta}{\alpha+\beta} \frac{-i * w_{max} * Et}{p} di = 0.$$

Let us simplify the equation now:

$$\frac{\alpha}{\alpha+\beta}Et[i]_0^{\gamma} - \frac{\beta}{\alpha+\beta}\frac{w_{max} * Et}{p}\left[\frac{i^2}{2}\right]_{\gamma}^{1} = 0$$

$$\frac{Et}{\alpha+\beta}\left(\alpha[i]_0^{\gamma} - \beta\frac{w_{max}}{p}\left[\frac{i^2}{2}\right]_{\gamma}^{1}\right) = 0$$

$$\alpha\gamma - \beta\frac{w_{max}}{p}\frac{1}{2}(1-\gamma^2) = 0$$

We also know that $w_\gamma = p$ so $w_\gamma = \gamma * w_{max} = p$. Thus,

$$\alpha\frac{p}{w_{max}} - \beta\frac{w_{max}}{p}\frac{1}{2}\left(1-\left(\frac{p}{w_{max}}\right)^2\right) = 0$$

$$\alpha\frac{p}{w_{max}} = \beta\frac{w_{max}}{p}\frac{1}{2}\frac{w_{max}^2 - p^2}{w_{max}^2}$$

$$\frac{\alpha}{\beta}\frac{p}{w_{max}} = \frac{1}{2p}\frac{w_{max}^2 - p^2}{w_{max}}.$$

Multiplying both sides by non-zero $p$ and further simplifying:

$$\frac{\alpha}{\beta}p^2 = \frac{1}{2}(w_{max}^2 - p^2)$$

$$\frac{\alpha}{\beta}p^2 + \frac{1}{2}p^2 = \frac{1}{2}w_{max}^2$$

$$\left(2\frac{\alpha}{\beta}+1\right)p^2 = w_{max}^2$$

$$p^2 = \frac{w_{max}^2}{\frac{2\alpha+\beta}{\beta}} = \frac{\beta}{2\alpha+\beta}w_{max}^2$$

$$p = w_{max}\sqrt{\frac{\beta}{2\alpha+\beta}},$$

by rejecting the negative root. That would also mean that

$$\gamma = \frac{p}{w_{max}} = \sqrt{\frac{\beta}{2\alpha+\beta}}.$$

The only endogenous value left in the utility is $Q$ so let us find it.

$$Q = average\ quality\ of\ character = \frac{1}{1-0}\int_0^1 character\ quality_i\, di$$

As before, split the population:

$$Q = \int_0^\gamma character\ quality_i\, di + \int_\gamma^1 character\ quality_i\, di$$

The character quality of workers would be the amount they have gained from trade, $-q_{ti}$. Note that the amount is positive since their value of $q_{ti}$ are negative. In the meantime, the virtual goods that the gamers have not sold is $Et - q_{ti}$, since they have spent $Et$ effort on playing. Therefore,

$$Q = \int_0^\gamma Et - q_{ti}\, di + \int_\gamma^1 -q_{ti}\, di.$$

Let us simplify the equation:

$$Q = \int_0^\gamma Et - q_{ti}\, di + \int_\gamma^1 -q_{ti}\, di$$

$$Q = \int_0^\gamma Et\, di + \int_0^\gamma -q_{ti}\, di + \int_\gamma^1 -q_{ti}\, di$$

$$Q = \int_0^\gamma Et\, di - \int_0^1 q_{ti}\, di$$

We know, however, that the last term is the amounts bought and sold on the market, therefore is zero. Thus,

$$Q = \int_0^\gamma Et\, di = Et[i]_0^\gamma = \gamma * Et = Et\sqrt{\frac{\beta}{2\alpha+\beta}}.$$

## Population values

Recall the solution to an individual's problem:

$$\begin{cases} \begin{cases} e_{wi} = 0 \\ q_{ti} = \frac{\alpha}{\alpha+\beta} Et \end{cases}, & if\ w_i < p \\ \begin{cases} e_{wi} = Et \\ q_{ti} = \frac{\beta}{\alpha+\beta}\frac{-w_i * Et}{p} \end{cases}, & if w_i > p \end{cases}.$$

Now, insert substitute the optimal values into the utility formula:

$$U_i = \begin{cases} \left(p * \frac{\alpha}{\alpha+\beta} Et\right)^\alpha \left(\frac{Et - \frac{\alpha}{\alpha+\beta} Et}{Q}\right)^\beta, & if\ w_i < p \\ \left(w_i * Et + p * \frac{\beta}{\alpha+\beta}\frac{-w_i * Et}{p}\right)^\alpha \left(\frac{Et - Et - \frac{\beta}{\alpha+\beta}\frac{-w_i * Et}{p}}{Q}\right)^\beta, & if\ w_i > p \end{cases}.$$

Let us simplify the expressions:

$$U_i = \begin{cases} Et^{\alpha+\beta}Q^{-\beta}\left(p * \frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(1-\frac{\alpha}{\alpha+\beta}\right)^{\beta}, & if\ w_i < p \\ Et^{\alpha+\beta}Q^{-\beta}\left(w_i + p * \frac{\beta}{\alpha+\beta}\frac{-w_i}{p}\right)^{\alpha}\left(1-1-\frac{\beta}{\alpha+\beta}\frac{-w_i * 1}{p}\right)^{\beta}, & if\ w_i > p \end{cases}$$

$$U_i = \begin{cases} Et^{\alpha+\beta}Q^{-\beta}\left(p * \frac{\alpha}{\alpha+\beta}\right)^{\alpha}\left(\frac{\alpha+\beta-\alpha}{\alpha+\beta}\right)^{\beta}, & if\ w_i < p \\ Et^{\alpha+\beta}Q^{-\beta}\left(w_i - w_i\frac{\beta}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\frac{w_i}{p}\right)^{\beta}, & if\ w_i > p \end{cases}$$

$$U_i = \begin{cases} Et^{\alpha+\beta}Q^{-\beta}p^{\alpha}\frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}}, & if\ w_i < p \\ Et^{\alpha+\beta}Q^{-\beta}w_i^{\alpha+\beta}\frac{1}{p^{\beta}}\left(\frac{\alpha+\beta-\beta}{\alpha+\beta}\right)^{\alpha}\left(\frac{\beta}{\alpha+\beta}\right)^{\beta}, & if\ w_i > p \end{cases}$$

$$U_i = \begin{cases} Et^{\alpha+\beta}Q^{-\beta}p^{\alpha}\frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}}, & if\ w_i < p \\ Et^{\alpha+\beta}Q^{-\beta}w_i^{\alpha+\beta}p^{-\beta}\frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}}, & if\ w_i > p \end{cases}.$$

Also note that the conditions $w_i \lesseqgtr p$ are equivalent to $i \lesseqgtr \gamma$**.** In addition, assume $\alpha + \beta = 1$; that does not decrease the realism of the model significantly, but reduces unnecessary calculations. Thus, the utility takes form

$$U_i = \begin{cases} EtQ^{-\beta}\alpha^{\alpha}\beta^{\beta}p^{\alpha}, & if\ i < \gamma \\ EtQ^{-\beta}\alpha^{\alpha}\beta^{\beta}w_i p^{-\beta}, & if\ \ i > \gamma \end{cases}.$$

Note that if the utilities are equal, $p^{\alpha} = w_i p^{-\beta} \Leftrightarrow p = w_i \Leftrightarrow i = \gamma$ so the medium person is truly indifferent so the utility function is continuous.

It is possible to make further substitutions of $Q$ and $\gamma$ into the utility formula:

$$U_i = \begin{cases} Et\left(Et\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{-\beta}\alpha^{\alpha}\beta^{\beta}p^{\alpha}, & if\ i < \sqrt{\frac{\beta}{2\alpha+\beta}} \\ Et\left(Et\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{-\beta}\alpha^{\alpha}\beta^{\beta}w_i\left(w_{max}\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{-\beta}, & if\ \ i > \sqrt{\frac{\beta}{2\alpha+\beta}} \end{cases}$$

$$= \begin{cases} Et^{\alpha}\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{-\beta}\alpha^{\alpha}\beta^{\beta}\left(w_{max}\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha}, & if\ i < \sqrt{\frac{\beta}{2\alpha+\beta}} \\ Et^{\alpha}\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{-\beta}\alpha^{\alpha}\beta^{\beta}w_i\left(w_{max}\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{-\beta}, & if\ \ i > \sqrt{\frac{\beta}{2\alpha+\beta}} \end{cases}$$

$$= \begin{cases} Et^{\alpha}\left(\sqrt{\dfrac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \alpha^{\alpha}\beta^{\beta}(w_{max})^{\alpha}, & if\ i < \sqrt{\dfrac{\beta}{2\alpha+\beta}} \\ Et^{\alpha}\left(\dfrac{\beta}{2\alpha+\beta}\right)^{-\beta} \alpha^{\alpha}\beta^{\beta} w_i (w_{max})^{-\beta}, & if\ \ i > \sqrt{\dfrac{\beta}{2\alpha+\beta}} \end{cases}$$

$$= \begin{cases} Et^{\alpha}\left(\sqrt{\dfrac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \alpha^{\alpha}\beta^{\beta}(w_{max})^{\alpha}, & if\ i < \sqrt{\dfrac{\beta}{2\alpha+\beta}} \\ Et^{\alpha}\left(\dfrac{1}{2\alpha+\beta}\right)^{-\beta} \alpha^{\alpha} w_i (w_{max})^{-\beta}, & if\ \ i > \sqrt{\dfrac{\beta}{2\alpha+\beta}} \end{cases}$$

$$= \begin{cases} Et^{\alpha}\left(\sqrt{\dfrac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \alpha^{\alpha}\beta^{\beta}(w_{max})^{\alpha}, & if\ i < \sqrt{\dfrac{\beta}{2\alpha+\beta}} \\ Et^{\alpha}\left(\dfrac{1}{2\alpha+\beta}\right)^{-\beta} \alpha^{\alpha} i * (w_{max})^{1-\beta}, & if\ \ i > \sqrt{\dfrac{\beta}{2\alpha+\beta}} \end{cases}$$

$$= \begin{cases} Et^{\alpha}\left(\sqrt{\dfrac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \alpha^{\alpha}\beta^{\beta}(w_{max})^{\alpha}, & if\ i < \sqrt{\dfrac{\beta}{2\alpha+\beta}} \\ Et^{\alpha}(2\alpha+\beta)^{\beta}\alpha^{\alpha} * i * w_{max}^{\alpha}, & if\ \ i > \sqrt{\dfrac{\beta}{2\alpha+\beta}} \end{cases}$$

Let us now calculate the total population utility:

$$TU = \int_0^1 U_i\, di = \int_0^{\gamma} Et^{\alpha}\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \alpha^{\alpha}\beta^{\beta}(w_{max})^{\alpha}\, di + \int_{\gamma}^{1} Et^{\alpha}(2\alpha+\beta)^{\beta}\alpha^{\alpha} * i * w_{max}^{\alpha}\, di$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}\left(\int_0^{\gamma}\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \beta^{\beta}\, di + \int_{\gamma}^{1}(2\alpha+\beta)^{\beta} * i\, di\right)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}\left(\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \beta^{\beta}\int_0^{\gamma} 1\, di + (2\alpha+\beta)^{\beta}\int_{\gamma}^{1} i\, di\right)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}\left(\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \beta^{\beta}[i]_0^{\gamma} + (2\alpha+\beta)^{\beta}\left[\frac{i^2}{2}\right]_{\gamma}^{1}\right)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}\left(\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta}\beta^{\beta}(\gamma-0)+\frac{1}{2}(2\alpha+\beta)^{\beta}(1-\gamma^2)\right)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}\left(\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta}\beta^{\beta}\sqrt{\frac{\beta}{2\alpha+\beta}}+\frac{1}{2}(2\alpha+\beta)^{\beta}\left(1-\sqrt{\frac{\beta}{2\alpha+\beta}}^{2}\right)\right)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}\left(\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta+1}\beta^{\beta}+\frac{1}{2}(2\alpha+\beta)^{\beta}\left(1-\frac{\beta}{2\alpha+\beta}\right)\right)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}\left(\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{2\alpha}\beta^{\beta}+\frac{1}{2}(2\alpha+\beta)^{\beta}\left(\frac{2\alpha+\beta-\beta}{2\alpha+\beta}\right)\right)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}\left(\left(\frac{\beta}{2\alpha+\beta}\right)^{\alpha}\beta^{\beta}+\frac{1}{2}(2\alpha+\beta)^{\beta-1}\left(\frac{2\alpha}{1}\right)\right)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}\left((2\alpha+\beta)^{-\alpha}\beta^{\alpha+\beta}+(2\alpha+\beta)^{\beta-1}\alpha\right)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}((2\alpha+\beta)^{-\alpha}\beta+(2\alpha+\beta)^{-\alpha}\alpha)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}(2\alpha+\beta)^{-\alpha}(\beta+\alpha)$$

$$= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}(2\alpha+\beta)^{-\alpha}$$

# Appendix 2: Full Solution of Problem without Trade

## Solution for individual's maximization problem

Recall that in the problem with trade, an individual is faced with the following problem:

$$\max_{e_{wi},q_{ti}} U_i = (w_i * e_{wi} + p * q_{ti})^{\alpha} \left(\frac{Et - e_{wi} - q_{ti}}{Q_{-i}}\right)^{\beta}.$$

If the trade is banned, nobody would be able to buy or sell in-game goods, so $q_{ti} = 0$ for all $i$. Thus, in the market without trade, an individual is faced with

$$\max_{e_{wi}} U_i = (w_i * e_{wi})^{\alpha} \left(\frac{Et - e_{wi}}{Q_{-i}}\right)^{\beta}.$$

Let us solve this one-variable optimization using calculus:

$$\frac{dU_i}{de_{wi}} = w_i \alpha (w_i * e_{wi})^{\alpha-1} \left(\frac{Et - e_{wi}}{Q_{-i}}\right)^{\beta} - \beta \frac{1}{Q_{-i}} (w_i * e_{wi})^{\alpha} \left(\frac{Et - e_{wi}}{Q_{-i}}\right)^{\beta-1} = 0$$

$$w_i \frac{\alpha}{\beta} = \frac{1}{Q_{-i}} \frac{w_i * e_{wi}}{\left(\frac{Et - e_{wi}}{Q_{-i}}\right)}$$

$$w_i \frac{\alpha}{\beta} = \frac{w_i * e_{wi}}{Et - e_{wi}}$$

$$w_i \frac{\alpha}{\beta}(Et - e_{wi}) = w_i * e_{wi}$$

$$w_i \frac{\alpha}{\beta} Et = w_i \frac{\alpha}{\beta} e_{wi} + w_i * e_{wi}$$

$$\frac{\alpha}{\beta} Et = \left(\frac{\alpha}{\beta} + 1\right) e_{wi}$$

$$e_{wi} = \frac{\frac{\alpha}{\beta} Et}{\left(\frac{\alpha}{\beta} + 1\right)} = \frac{\alpha}{\alpha + \beta} Et.$$

Note that $w_i$ was factored out during the process. Thus, we have a selection of effort, independent from the wage; thus, all the individuals would do the same decision. Also note that the resulting $e_{wi} \in (0, Et)$ so we have an interior solution. As the original equation is in Cobb-Douglas form, we get a point of maximum utility. Therefore, that is a solution to individual's problem.

## Solving for population values

We do not have to consider the trade market because it does not exist. We can solve for $Q_{-i}$, however. As before, big population allows us to say that $Q_{-i} = Q = avarage\ character\ quality$ for all $i$. Each individual gets $g_i = e_{pi} = Et - e_{wi}$, so

$$Q = Et - e_{wi} = Et - \frac{\alpha}{\alpha + \beta} Et = \frac{\beta}{\alpha + \beta} Et,$$

as average would be the same as each individual's decision (recall that all the individuals select the same $e_{wi}$).

In that case, each individual would get a utility of

$$U_i = (w_i * e_{wi})^\alpha \left(\frac{Et - e_{wi}}{Q}\right)^\beta = \left(w_i * \frac{\alpha}{\alpha+\beta} Et\right)^\alpha \left(\frac{Et - e_{wi}}{Et - e_{wi}}\right)^\beta = \left(w_i * \frac{\alpha}{\alpha+\beta} Et\right)^\alpha.$$

Note that the utility from playing had factored out – that happens because everyone gets the same quality of characters.

Thus, the total population utility would be

$$TU = \int_0^1 U_i\, di = \int_0^1 \left(w_i * \frac{\alpha}{\alpha+\beta} Et\right)^\alpha di = \left(\frac{\alpha}{\alpha+\beta} Et\right)^\alpha \int_0^1 (i * w_{max})^\alpha di = \left(w_{max} \frac{\alpha}{\alpha+\beta} Et\right)^\alpha \left[\frac{i^{\alpha+1}}{\alpha+1}\right]_0^1$$
$$= \frac{1}{\alpha+1}\left(w_{max} \frac{\alpha}{\alpha+\beta} Et\right)^\alpha [1-0] = \frac{1}{2\alpha+\beta}\left(w_{max} \frac{\alpha}{\alpha+\beta} Et\right)^\alpha.$$

## Comparison with the version with trade

Let us compare $Q$ – the average player's quality, remembering that $\alpha, \beta \in [0,1]$ and assuming $\alpha + \beta = 1$.

$$\begin{aligned}
Q_{trade} &> Q_{no\ trade} \\
Et\sqrt{\frac{\beta}{2\alpha+\beta}} &> Et\frac{\beta}{\alpha+\beta} \\
\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^2 &> \left(\frac{\beta}{\alpha+\beta}\right)^2 \\
\frac{\beta}{2\alpha+\beta} &> \frac{\beta^2}{(\alpha+\beta)^2} \\
\frac{1}{\alpha+(\alpha+\beta)} &> \frac{\beta}{1} \\
\frac{1}{\alpha+1} &> \frac{\beta}{1} \\
1 &> \beta(\alpha+1) \\
1 &> (1-\alpha)(1+\alpha) \\
1 &> (1-\alpha^2) \\
0 &> -\alpha^2 \\
\alpha^2 &> 0 \\
\alpha &> 0
\end{aligned}$$

Thus, $Q_{trade} > Q_{no\ trade}$ for $\alpha > 0$, since $\alpha < 0$ gets rejected.

Let us compare the utility of 0th individual.

$$U_{trade,0} > U_{no\ trade,0}$$
$$Et^\alpha \left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \alpha^\alpha \beta^\beta (w_{max})^\alpha > \left(w_0 * \frac{\alpha}{\alpha+\beta} Et\right)^\alpha$$
$$Et^\alpha \left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta} \alpha^\alpha \beta^\beta (w_{max})^\alpha > 0,$$

as all terms on the left are positive and $w_0 = 0$ as $w_i = i * w_{max}$. Thus, 0th individual is better off with trade.

Let us compare the utility of 1st individual.

$$\begin{aligned}
U_{trade,1} &> U_{no\ trade,1} \\
Et^{\alpha}(2\alpha+\beta)^{\beta}\alpha^{\alpha} * i * w_{max}^{\alpha} &> \left(w_1 * \frac{\alpha}{\alpha+\beta}Et\right)^{\alpha} \\
Et^{\alpha}(2\alpha+\beta)^{\beta}\alpha^{\alpha} * 1 * w_{max}^{\alpha} &> (1 * w_{max})^{\alpha} * \frac{\alpha^{\alpha}}{1}Et^{\alpha} \\
(2\alpha+\beta)^{\beta} * w_{max}^{\alpha} &> w_{max}^{\alpha} \\
(2\alpha+\beta)^{\beta} &> 1 \\
\big(\alpha+(\alpha+\beta)\big)^{\beta} &> 1 \\
(\alpha+1)^{\beta} &> 1 \\
(\alpha+1) &> 1 \\
\alpha &> 0,
\end{aligned}$$

thus, the 1st individual would be better off with trade if $\alpha > 0$.

Let us compare the utility of $\gamma^{th}$ individual.

$$\begin{aligned}
U_{trade,\gamma} &< U_{no\ trade,\gamma} \\
Et^{\alpha}\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta}\alpha^{\alpha}\beta^{\beta}(w_{max})^{\alpha} &< \left(w_{\gamma} * \frac{\alpha}{\alpha+\beta}Et\right)^{\alpha} \\
Et^{\alpha}\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta}\alpha^{\alpha}\beta^{\beta}(w_{max})^{\alpha} &< \gamma^{\alpha} * w_{max}^{\alpha} * \frac{\alpha^{\alpha}}{1}Et^{\alpha} \\
\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta}\beta^{\beta} &< \gamma^{\alpha} \\
\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta}\beta^{\beta} &< \sqrt{\frac{\beta}{2\alpha+\beta}}^{\alpha} \\
\sqrt{\frac{\beta}{2\alpha+\beta}}^{-\beta}\beta^{\beta} &< 1 \\
\sqrt{\frac{2\alpha+\beta}{\beta}}\beta &< 1^{\frac{1}{\beta}} \\
\sqrt{\big(\alpha+(\alpha+\beta)\big)\beta} &< 1 \\
\sqrt{(\alpha+1)(1-\alpha)} &< 1 \\
\sqrt{1-\alpha^2} &< 1 \\
1-\alpha^2 &< 1 \\
0 &< \alpha.
\end{aligned}$$

Therefore, $\gamma^{th}$ individual would be better off without trade.

Let us now compare the total utilities:

$$
\begin{aligned}
TU_{trade} &> TU_{no\ trade} \\
Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}(2\alpha+\beta)^{-\alpha} &> \frac{1}{2\alpha+\beta}\left(w_{max}\frac{\alpha}{\alpha+\beta}Et\right)^{\alpha} \\
Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}(2\alpha+\beta)^{-\alpha} &> (2\alpha+\beta)^{-1}w_{max}^{\alpha}\left(\frac{\alpha}{\alpha+\beta}\right)^{\alpha}Et^{\alpha} \\
\alpha^{\alpha}(2\alpha+\beta)^{-\alpha} &> (2\alpha+\beta)^{-1}\frac{\alpha^{\alpha}}{1} \\
(2\alpha+\beta)^{-\alpha+1} &> 1 \\
(2\alpha+\beta)^{\beta} &> 1 \\
(1+\alpha)^{\beta} &> 1^{\beta} \\
1+\alpha &> 1 \\
\alpha &> 0
\end{aligned}
$$

Thus, if $\alpha > 0$, the people are better off with trade.

## Additional results

It is also possible to calculate the individuals that are indifferent between banning and not banning the trade. In the graph, those are located at the intersection of the two utility curves. By identifying such individuals, we can find the band of individuals that prefer the ban to no ban – they will be in between of the indifferent individuals. By equating the utilities for the two problems, it is possible to get that

$$i_{indiff,1} = \left(\sqrt{\frac{\beta}{2\alpha+\beta}}^{\,\alpha-\beta}\beta^{\beta}\right)^{\frac{1}{\alpha}}$$

$$i_{indiff,2} = \frac{1}{2\alpha+\beta} = \frac{1}{\alpha+1}$$

Those are actually both decreasing in $\alpha$ and the band gets bigger as $\alpha$ increases. It is better depicted visually in the Figure 4 Individuals who are better off without trade (colored) on page 6.

It is also possible to extend the feasible region diagram from page 16 with the solution to the problem without trade and the corner solutions:

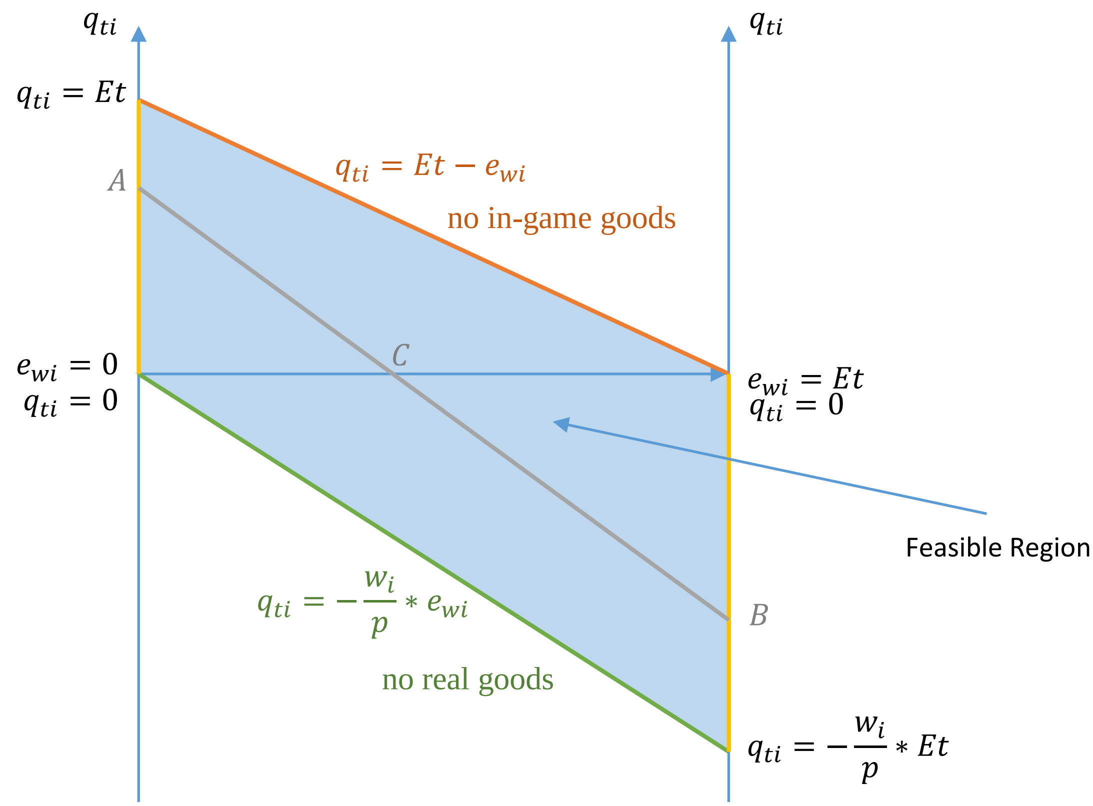


*Figure 7 Feasible region for an individual's decision with optimum points*

with $A$: $\{e_{wi} = 0, q_{ti} = \frac{\alpha}{\alpha+\beta}Et\}$ – a gamer's solution, $B$: $\{e_{wi} = Et, q_{ti} = \frac{\beta}{\alpha+\beta}\frac{-w_i}{p}Et\}$ – a worker's solution, $C$: $\{e_{wi} = \frac{\alpha}{\alpha+\beta}Et, q_{ti} = 0\}$ – solution for a problem with no trade ($q_{ti} = 0$). Observe that whose three points can be characterized by a line (shown in grey)

$$q_{ti} = \frac{\alpha}{\alpha+\beta}Et - \frac{-w_i}{p}e_{wi}.$$

In addition, it is possible to derive that that line from a system

$$\begin{cases} \frac{\alpha}{\beta}w_i = \frac{(w_i * e_{wi} + p * q_{ti})}{Et - e_{wi} - q_{ti}} \\ (w_i - p) = 0 \end{cases},$$

that is, the solution for the two-variable calculus optimization. (Note however, that the solution restricts $w_i = p$). Thus, the diagram shows that all the solutions derived are the solutions to the same problem with the points $A$, $B$ and $C$ being restricted to $e_{wi} = 0$, $e_{wi} = Et$ and $q_{ti} = 0$ respectfully.

## Appendix 3: Numeric Solutions

Here we are assigning values to the exogenous variables and see what numeric values are created.

First of all, it was selected that $Et = 8$. That number was chosen as that is usually the number of hours a person works. Then $w_{max}$ =37.97$ – the average per hour Canadian salary for a management position (Average hourly wages of employees by selected characteristics and occupation, unadjusted data, by province (monthly), 2014). It is expected that the people with the higher income would not probably play online games. In addition, the average salary would be 18.985$ which falls in between of 15-24 years individuals' salary (14.01$) and 25-54 years individual's salary (26.01$) from the same source. Further, $\alpha = 0.4$ to allow individuals to prefer virtual characters slightly more than the real goods. Then, those values were inserted into the formulas derived in the paper.

$Et = 8$
$w_{max} = 37.97\$$
$\alpha = 0.4$

| | With Trade | No Trade |
|---|---|---|
| $e_{wi}$ | 0 for players, 8 for workers | 3.2 hours |
| $Q$ | 5.237 | 4.8 |
| $\gamma$ | 0.6547 | - |
| $p$ | 24.86 | - |
| $U_i$ | $\begin{cases} 5.464 & i < 0.6547 \\ 8.347\,i & otherwise \end{cases}$ | $6.821i^{0.4}$ |

As we can see, the individuals do have lower character quality for the version with no trade. The plot of utility functions is the following:

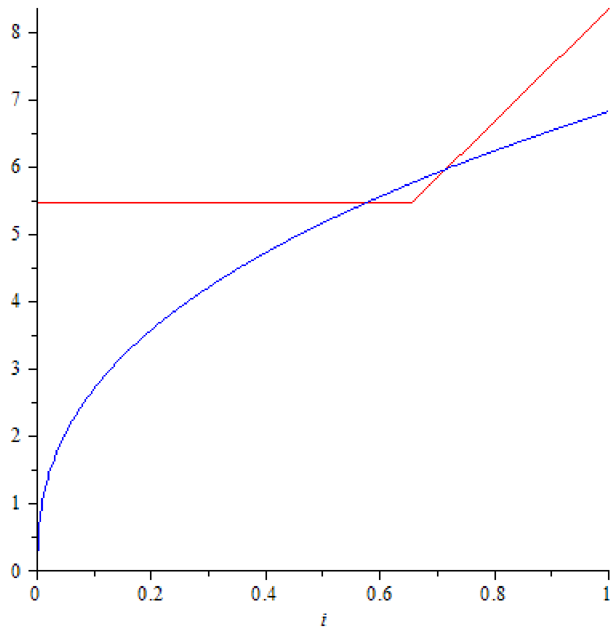


*Figure 8 Utility functions for trade (red) and no trade (blue) under* $\alpha = 0.4$

Thus, we see the picture that looks the same as the generic one from section Comparison of the solutions for the two versions on page 8.

Here is a plot for $\alpha = 0.7$, a situation when people value the real goods more:

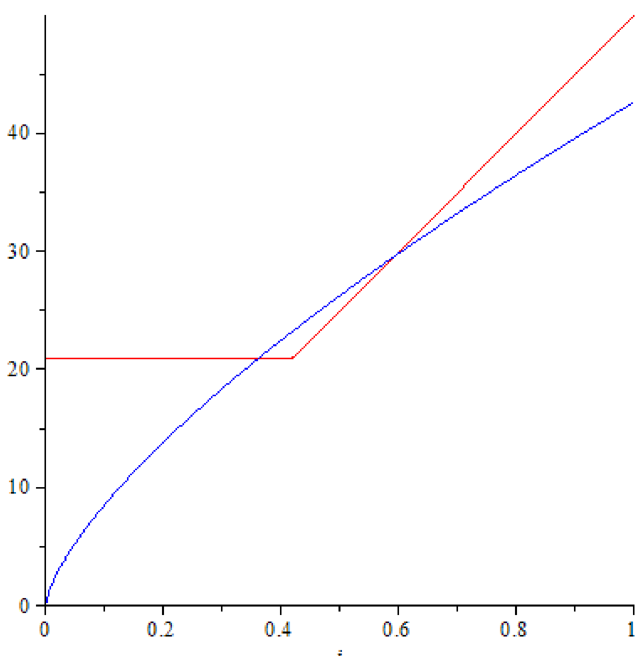


*Figure 9 Utility functions for trade (red) and no trade (blue) under* $\alpha = 0.7$

We see an increase in the number of workers and the people who would benefit from the ban of the trade. The number of workers increase because people are less willing to give up a revenue from work and less willing to buy virtual goods, therefore $\gamma$ would decrease. Also note that the high-income individuals are not as better off with trade as they are under $\alpha = 0.4$. Also, let us compare the numeric values:

$Et = 8$
$w_{max} = 37.97\$$

| | $\alpha = 0.4$ (Prefer virtual) | | $\alpha = 0.7$ (Prefer real) | |
|---|---|---|---|---|
| | With Trade | No Trade | With Trade | No Trade |
| $e_{wi}$ | 0 for players, 8 for workers | 3.2 hours | 0 for players, 8 for workers | 5.6 hours |
| $Q$ | 5.237 | 4.8 | 3.360 | 2.4 |
| $\gamma$ | 0.6547 | - | 0.4201 | - |
| $p$ | 24.86 | - | 15.95 | - |

We can see that the individuals do work more under the no trade version ($5.6 > 3.2$) and that the character qualities are smaller than when people prefer real more. Those effect make sense since the individuals prefer to spend more on the real goods as $\alpha$ is bigger. However, note that when people prefer real more, $Q$ rises much higher when going from no trade to trade ($\frac{3.360-2.4}{2.4} * 100\% = 40\%$ increase) compared to the situation when $\alpha = 0.4$ ($\frac{5.237-4.8}{4.8} * 100\% = 9{,}1\%$ increase). The explanation for the difference would that the trade equilibrium with $\alpha = 0.7$ makes people to work more than in the $\alpha = 0.4$ equilibrium so less virtual goods are produced. Thus, a trade ban would be less efficient when people want virtual goods.

## Appendix 4: Extension: The Gaming Company enters the virtual market

The situation when a gaming company interferes the market is not uncommon. For instance, the company Blizzard Entertainment had recently announced a service of leveling up a hero up to 90th level (while the maximum level is going to be 100th) (Wesley, 2014). In that case, the gamers would prefer to buy the good from company first as it is easier and more safe than make a purchases on a non-official market.

In the case if company releases a positive amount $F$ of virtual good to the market at the market price $p$, it would distort the virtual market equation:

$$\int_0^{\gamma} q_{ti}\, di + \int_{\gamma}^{1} q_{ti}\, di + F = 0,$$

given that $\int_{\gamma}^{1} q_{ti}\, di \geq -F$; that is, if F does not overly satisfy the buyers. Note that it is almost costless for a company to create the virtual goods as that includes only manipulation with the programs.

The company might also use a profit-maximizing quantity, although Blizzard (the developers of WoW) claim not to set it so as Wesley quotes (2014). Therefore, a company might do

$$\max p * F,$$

where p is a market price. Let us see what happens to the market:

$$\int_0^{\gamma} q_{ti}\, di + \int_{\gamma}^{1} q_{ti}\, di + F = 0$$

$$\int_0^{\gamma} \frac{\alpha}{\alpha+\beta} Et\, di + \int_{\gamma}^{1} \frac{\beta}{\alpha+\beta} \frac{-i * w_{max} * Et}{p} di + F = 0$$

$$\frac{Et}{\alpha+\beta}\left(\alpha[i]_0^{\gamma} - \beta \frac{w_{max}}{p}\left[\frac{i^2}{2}\right]_{\gamma}^{1}\right) + F = 0$$

$$\frac{Et}{\alpha+\beta}\left(\alpha(\gamma) - \frac{\beta}{2}\frac{w_{max}}{p}(1-\gamma^2)\right) + F = 0$$

$$Et\left(\alpha\left(\frac{p}{w_{max}}\right) - \frac{\beta}{2}\frac{w_{max}}{p}\left(1 - \frac{p}{w_{max}}^{2}\right)\right) + F = 0$$

$$Et\left(\alpha\left(\frac{p}{w_{max}}\right) - \frac{\beta}{2}\frac{w_{max}}{p} + \frac{\beta}{2}\frac{w_{max}}{p}\frac{p^2}{w_{max}^2}\right) + F = 0$$

$$Et\left(\alpha\left(\frac{p}{w_{max}}\right) - \frac{\beta}{2}\frac{w_{max}}{p} + \frac{\beta}{2}\frac{p}{w_{max}}\right) + F = 0$$

$$Et\left(2\alpha\left(\frac{p^2}{w_{max}}\right) - \beta w_{max} + \beta\frac{p^2}{w_{max}}\right) + 2Fp = 0$$

$$Et\left((2\alpha+\beta)\left(\frac{p^2}{w_{max}}\right) - \beta w_{max} + \frac{2Fp}{Et}\right) = 0$$

$$\frac{(2\alpha+\beta)}{w_{max}} p^2 + \frac{2F}{Et} p - \beta w_{max} = 0$$

$$p_{1,2}=\frac{-\frac{2F}{Et}\pm\sqrt{\left(\frac{2F}{Et}\right)^2-4\frac{(2\alpha+\beta)}{w_{max}}(-\beta w_{max})}}{2\frac{(2\alpha+\beta)}{w_{max}}}$$

$$p_{1,2}=\frac{-\frac{2F}{Et}\pm\sqrt{\frac{4F^2}{Et^2}+4\frac{(2\alpha+\beta)}{w_{max}}\beta w_{max}}}{2\frac{(2\alpha+\beta)}{w_{max}}}$$

$$p_{1,2}=\frac{-2\frac{F}{Et}\pm 2\sqrt{\frac{F^2}{Et^2}+(2\alpha+\beta)\beta}}{2\frac{(2\alpha+\beta)}{w_{max}}}$$

$$p_{1,2}=\frac{-\frac{F}{Et}\pm\sqrt{\frac{F^2}{Et^2}+(1+\alpha)(1-\alpha)}}{\frac{(2\alpha+\beta)}{w_{max}}}$$

$$p_{1,2}=\frac{-\frac{F}{Et}\pm\sqrt{\frac{F^2}{Et^2}+1-\alpha^2}}{\frac{(2\alpha+\beta)}{w_{max}}}$$

$$p=\frac{-\frac{F}{Et}+\sqrt{\frac{F^2}{Et^2}+1-\alpha^2}}{\frac{(2\alpha+\beta)}{w_{max}}},$$

since the root with a minus is negative for sure and the root with plus is positive, since

$$\sqrt{\frac{F^2}{Et^2}+1-\alpha^2}\geq\sqrt{\frac{F^2}{Et^2}}=\frac{F}{Et}.$$

Let us substitute the price into the maximization expression, take a derivative using technology and set it to 0:

$$\max\frac{-\frac{F}{Et}+\sqrt{\frac{F^2}{Et^2}+1-\alpha^2}}{\frac{(2\alpha+\beta)}{w_{max}}}*F=\frac{w_{max}}{2\alpha+\beta}F\left(-\frac{F}{Et}+\sqrt{\frac{F^2}{Et^2}+1-\alpha^2}\right)$$

$$\frac{d}{dF}:\frac{w_{max}}{(2\alpha+\beta)Et^2}\frac{\left(\sqrt{\frac{F^2}{Et^2}+1-\alpha^2}Et-F\right)^2}{\sqrt{\frac{F^2}{Et^2}+1-\alpha^2}}=0$$

$$\sqrt{\frac{F^2}{Et^2}+1-\alpha^2}Et-F=0$$

$$\sqrt{\frac{F^2}{Et^2}+1-\alpha^2}=\frac{F}{Et}$$

However, that holds iff $\alpha = 1$. Otherwise, the derivative is always positive so the firm sets F as big as possible. That is, the whole demand of workers: $\int_{\gamma}^{1} q_{ti}\, di = F$ and so $\int_{0}^{\gamma} q_{ti}\, di = 0$ (from $\int_{0}^{\gamma} q_{ti}\, di + \int_{\gamma}^{1} q_{ti}\, di + F = 0$). The equality $\int_{0}^{\gamma} q_{ti}\, di = 0$ can only happen if $\gamma = 0$ which is equivalent to the firm to be the only supplier to the market. That would mean, in turn that the firm is no longer a price-taker so it also choses the profit-maximizing price facing a demand of

$$\int_{0}^{1} q_{ti}\, di = \frac{\beta}{\alpha+\beta}\frac{w_{max} * Et}{p}\left[\frac{i^2}{2}\right]_0^1 = \frac{\beta}{\alpha+\beta}\frac{w_{max} * Et}{p}\frac{1}{2}.$$

Thus the firm's problem is

$$\max_{p} \frac{\beta}{\alpha+\beta}\frac{w_{max} * Et}{p}\frac{1}{2} * p = \frac{\beta}{\alpha+\beta}\frac{w_{max} * Et}{2},$$

so, it can select any price as it has $\frac{\beta}{\alpha+\beta}\frac{w_{max}*Et}{2}$ of guaranteed profit. Moreover, $Q = F = \frac{\beta}{\alpha+\beta}\frac{w_{max}*Et}{p}\frac{1}{2}$ as there are no other producers. Recall from a solution for the original problem that the workers get $U_i = EtQ^{-\beta}\alpha^{\alpha}\beta^{\beta}w_i p^{-\beta}$.

Substitution of $Q$ gives

$$\begin{aligned} U_i &= EtQ^{-\beta}\alpha^{\alpha}\beta^{\beta}w_i p^{-\beta} \\ &= Et\left(\frac{\beta}{\alpha+\beta}\frac{w_{max} * Et}{p}\frac{1}{2}\right)^{-\beta}\alpha^{\alpha}\beta^{\beta}w_i p^{-\beta} \\ &= Et\left(\frac{1}{1}\frac{w_{max} * Et}{1}\frac{1}{2}\right)^{-\beta}\alpha^{\alpha}w_i \\ &= Et^{1-\beta}\left(\frac{1}{w_{max}^{\beta}}2^{\beta}\right)\alpha^{\alpha}w_i \\ &= Et^{1-\beta}2^{\beta}\alpha^{\alpha}i * w_{max}^{1-\beta} \\ &= Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}2^{\beta}i. \end{aligned}$$

Compare that with what the workers receive under the trade equilibrium:

$$\begin{gathered} U_{(firm,i)} \geq U_{trade-worker,i} \\ Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}2^{\beta}i \geq Et^{\alpha}(2\alpha+\beta)^{\beta}\alpha^{\alpha} * i * w_{max}^{\alpha} \\ 2^{\beta}? \geq (2\alpha+\beta)^{\beta} \\ 2 \geq (2\alpha+\beta) \\ 2 \geq (1+\alpha) \\ 1 \geq \alpha, \end{gathered}$$

thus the workers are better off with the firm is providing the virtual goods. That might happen because there is a negative relation between $Q$ and $p$ set by the profit-maximizing firm. If the $Q$ is high so bad for the workers, the $p$ is low which is good for them.

However, what about the gamers who used to sell in-game goods and have low income? We can check when they are better off, recalling the fact that for the gamers $i < \sqrt{\frac{\beta}{2\alpha+\beta}}$:

$$U_{(firm,i)} \geq U_{trade-gamer,i}$$

$$Et^{\alpha}\alpha^{\alpha}w_{max}^{\alpha}2^{\beta}i \geq Et^{\alpha}\left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta}\alpha^{\alpha}\beta^{\beta}(w_{max})^{\alpha}$$

$$2^{\beta}i \geq \left(\sqrt{\frac{\beta}{2\alpha+\beta}}\right)^{\alpha-\beta}\beta^{\beta}$$

$$i \geq (i)^{\alpha-\beta}\beta^{\beta}\frac{1}{2^{\beta}}$$

$$i^{1-(\alpha-\beta)} \geq \beta^{\beta}\frac{1}{2^{\beta}}$$

$$i^{2\beta} \geq \left(\frac{\beta}{2}\right)^{\beta}$$

$$i \geq \sqrt{\frac{\beta}{2}}$$

Thus, there would always be low-income individuals who are suffering. Note that if $\beta > \frac{1}{2}$, $i \geq \sqrt{\frac{\frac{1}{2}}{2}} = \frac{1}{2}$ so more than a half of individuals would be worse off! As $\beta$ is a parameter that shows how much utility people get from playing, we conclude that it would be terrible for them if the company decides to monopolize the market for in-game goods.

Note that the low-income individuals would have an incentive to switch to selling and undercut the monopoly price – in that case workers would buy from them first. In order to avoid that, the company can set really low price, $\epsilon \to 0$, for the virtual goods; recall that the firm can set any price it wants and get the same profit. Thus, the individuals would not switch to producing virtual goods as their jobs would pay more. In order to have such a low price, the firm would release a massive quantity of virtual goods therefore creating inflation in character quality, making it impossible for an individual to develop a satisfying character quality from effort put into playing. The only valid way would be to buy the virtual goods from the company.

In general, we have achieved here an equilibrium where no character quality is created by the players – everyone works $Et$ of effort and buys the character quality from the gaming company. Obviously, the people with higher income can afford to buy more in-game goods so they would be better in the game while the low-income individuals would suffer. They would suffer more the more they care about winning, that is, the higher is $\beta$. Thus, the company trying to maximizes the profit in in-game goods market resulted in producing a so-called "pay-to-win" games, the ones in which real-money input in the game matters how good you play and how much fun you would get. Note that the company forces you to pay to be good – there is no way to have high $Q$ if you are not buying. Although that situation can be an effect of chosen functional forms, the firm would always have an incentive to sell virtual goods, unless they care about the players to have ability to be able to get a competitive character without paying the money to the company. That is what happening in the Wesley news article; he states that "Blizzard didn't want to 'devalue the accomplishment of levelling'" as a reason for not setting a profit-maximizing price (Wesley, 2014). The model had shown to us what could happen otherwise.